

VARIATIONAL METHOD IN THE DESIGN OF AN OPTIMUM
SOLAR WATER HEATER STORAGE TANK

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*To my beloved wife, Suhaila, and my children,
Muhammad Syazwan and Muhammad Syahmi.*

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ABSTRACT

Solar energy is a renewable resource because it is non-exhaustible and is available in abundance. The average daily total solar radiation amount in Malaysia is about 4500 Wh/m^2 , K.S Ong (1994). Intermittency and non-availability at night are the main drawbacks to the use of solar energy. However, heat storage devices could be designed to store heat for up to 24 hours if necessary. Solar water heaters are now been accepted as a reliable source of providing hot water heating in many domestic homes and are becoming more popular. Unfortunately, solar water heaters are still considered luxurious items in Malaysia. Users will always seek the product with cheaper price but at the same time still fulfill their need. Thus, a cheaper and efficient solar water heater system is required to be designed. The only way to produce cheaper and efficient product is through the optimization process to obtain the minimum cost but still maintain the specification required or call as constraints. In this project the hot water storage tank will be analyzed to obtain the optimum cost. Thermosyphon-flow solar water heating system is preferred for obvious economic reasons since they do not require circulation pumps and control units. Average temperature of the hot water in the storage tank is determined through the temperature distribution simulation. The overall average temperature obtained is 49.3°C . The overall average temperature is used to solve the optimization problem. The constraints involve for the optimization are the tank volume and heat losses allowed from the water in the tank. The *Lagrange multiplier* method, which is based on derivatives of the objective function and the constraints are applied. The optimum independent variables and Lagrange multipliers is solved by computational approach. The minimum cost obtained for hot water storage tank with 225 l capacity is RM 1322.15. The Lagrange multipliers represent the sensitivity coefficient, which define as the rate of change of the objective function with the constraint at the optimum. This optimization method is very useful for the manufacturer in adjusting the design variables to come up with the final design.

ABSTRAK

Tenaga solar adalah sumber yang boleh diperbaharui kerana ianya tidak akan pupus dan boleh didapati dengan mudah. Jumlah purata harian radiasi tenaga solar di Malaysia adalah 4500 Wh/m^2 , K.S Ong (1994). Kehadiran tenaga solar yang tidak menentu dan ketiadaannya pada waktu malam adalah kelemahan utama penggunaan tenaga solar. Walaubagaimanapun, peralatan menyimpan haba boleh direkabentuk untuk menyimpan haba sehingga 24 jam jika perlu. Pada masa kini pemanas air solar diterima sebagai sumber untuk membekalkan air panas dalam kebanyakan rumah dan telah menjadi popular. Malangnya pemanas air solar masih dianggap sebagai peralatan mewah di Malaysia. Pengguna akan sentiasa mencari produk dengan harga yang rendah tetapi pada masa yang sama masih memenuhi keperluan mereka. Oleh itu, sistem pemanas air solar dengan harga yang lebih rendah dan cekap perlu dihasilkan. Ini boleh dilakukan hanya dengan melalui proses optimisasi untuk mendapatkan harga yang terendah tetapi masih mengekalkan spesifikasi yang diperlukan atau dikenali sebagai kekangan. Dalam projek ini, tangki menyimpan air panas akan dianalisis untuk memperoleh kos yang optimum. Aliran pemanasan semulajadi sistem pemanas air solar dipilih disebabkan ianya ekonomik kerana tidak memerlukan pam dan unit kawalan. Suhu purata air panas di dalam tangki ditentukan melalui simulasi pengagihan suhu. Suhu purata keseluruhan yang diperolehi adalah 49.3°C . Suhu purata keseluruhan digunakan untuk menyelesaikan masalah optimisasi. Kekangan yang terlibat untuk optimisasi ini adalah isipadu tangki dan haba yang hilang dari air dalam tangki. Kaedah *pekali Lagrange*, iaitu berdasarkan pembezaan fungsi objektif dan kekangan digunakan. Pemboleh ubah bebas optimum dan pekali Lagrange diselesaikan dengan menggunakan komputer. Kos minimum yang diperolehi untuk tangki menyimpan air panas dengan isipadu 225 l adalah RM 1322.15. Pekali Lagrange mewakili pekali sensitiviti, didefinisikan sebagai kadar perubahan fungsi objektif dengan kekangan pada keadaan yang optimum. Kaedah optimisasi ini adalah sangat berguna kepada pengeluar dalam mengubahsuai pemboleh ubah rekabentuk untuk menghasilkan rekabentuk akhir.

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LIST OF SYMBOLS

A	-	cross-sectional area of tank, m^2
C_p	-	specific heat of water, $kJ/kg.^{\circ}C$
D	-	price of mild steel, RM/kg
E	-	price of fiberglass wool, RM/kg
F	-	welding cost, RM/m
G_1	-	constraint 1, m^3
G_2	-	constraint 2, J/s
G_3	-	constraint 3, J/s
G_4	-	constraint 4, J/s
H_1	-	insulator thickness at the top of the tank, m
H_2	-	insulator thickness at the bottom of the tank, m
h	-	overall heat transfer coefficient, $W/m^2.K$
h_1	-	convective heat transfer coefficient of the water, $W/m^2.K$
h_2	-	convective heat transfer coefficient of the ambient air, $W/m^2.K$
k_1	-	thermal conductivity of the fiberglass wool, $W/m.K$
k_w	-	thermal conductivity of water, $W/m.K$
L	-	height of tank, m
m_w	-	mass of water, kg
P	-	perimeter of tank, m
Q	-	total heat losses from tank, J/s

Q_1	-	heat losses from sidewall of the tank, J/s
Q_2	-	heat losses from top of the tank, J/s
Q_3	-	heat losses from bottom of the tank, J/s
R_t	-	thermal resistance, K/W
r_1	-	radius of tank, m
r_2	-	radius of insulator, m
S_{c1}	-	sensitivity coefficient constraint 1, RM/m ³
S_{c2}	-	sensitivity coefficient constraint 2, RM/ J/s
S_{c3}	-	sensitivity coefficient constraint 3, RM/ J/s
S_{c4}	-	sensitivity coefficient constraint 4, RM/ J/s
T	-	water temperature in the tank, °C
T_t	-	top water temperature in the tank, °C
T_a	-	ambient temperature, °C
T_{avg}	-	average water temperature in the tank, °C
t_t	-	thickness of tank material, m
t_c	-	thickness of casing material, m
U	-	objective function, RM
U_{opt}	-	optimal value of U , RM
U_w	-	total internal energy, kJ
u_w	-	internal energy per mass, kJ/kg
V	-	volume of tank, m ³
w	-	average vertical water velocity in the tank, m/s
ρ_{ss}	-	density of tank material (mild steel), kg/m ³
ρ_i	-	density of insulator (fiberglass wool), kg/m ³
ρ_w	-	density of water, kg/m ³

τ	-	physical time, s
λ_1	-	Lagrange multiplier constraint 1
λ_2	-	Lagrange multiplier constraint 2
λ_3	-	Lagrange multiplier constraint 3
λ_4	-	Lagrange multiplier constraint 4

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CHAPTER 1

INTRODUCTION

1.1 Overview

The world depends on the availability of cheap and abundant energy for survival. The growth of modern industry and the explosive increase in world population have made additional and extensive demands on the world supply of energy. Industry requires fuel for power while the population requires fuel for food production. While the industrial economy might be able to withstand spiraling fuel costs there would come a time when supply would be insufficient to meet demand because of the increasing scarcity of fossil fuels. Fossil fuels like coal, oil and natural gas are grouped with nuclear fusion and nuclear fission power under non-renewable energy resources because they can be depleted and there are major obstacles to their utilization. We may have to seek alternative renewable sources of energy when the time comes. Energy resources like biomass, biogas, solar, wind, water and geothermal energy are grouped under renewable resources because they are non-exhaustible and are available in abundance.

Solar energy still remains a major and largely untapped sources of power. It is cheap, abundant, safe to use and no transportation cost required. It also has the added advantage of being pollution-free. Intermittency and non-availability at night are the main drawbacks to the use of solar energy. However, heat storage devices could be design to store heat for up to 24 hours if necessary. Large solar collector surface also are required because of the low-intensity nature of solar radiation. For high-temperature applications, mirrors could be used to concentrate the energy.

Solar water heaters are now been accepted as a reliable source of providing hot water heating in many domestic homes and are becoming more popular. In the early days, solar water heaters were considered a novelty. Householders then were skeptical about the ability of the solar heaters to perform satisfactorily to supply sufficient hot water for their daily needs. They are now willing to install solar systems in their homes.

Water heaters are still considered luxurious household items in Malaysia. However, with the rising standard of living, the trend is towards water heaters after air conditioners have been installed, especially in the urban communities. The rural communities and lower income group citizens would be more inclined to continue using cold water for bathing and domestic washing. It was thought previously that the middle income group would be the most likely potential user of solar water heaters. However, it was the higher income group that has provided the impetus for the solar water heater market. The demand for solar water heaters would depend upon public acceptability based on economics, aesthetics, reliability and the availability and cost of conventional fuel.

The consumption of solar water heater would be more widely if the manufacturer were able to offer the systems with lower price. Therefore, in order to produce the cheaper solar water heater with high efficiency the optimization of the design need to be done. This project will focus its attention on the design of an optimum solar water heater storage tank. Other components will not be considered further.

1.2 Literature Survey

According to K.S. Ong (1994), solar water heaters collect and convert incident solar radiation energy to useful thermal energy in the form of hot water. They essentially consist of solar collectors for absorbing and converting the solar

energy and an insulated storage vessel or tank to contain the heated water. There are basically two types of solar water heaters, which is the integrated collector-cum-storage type and the collector-coupled-to storage tank type. The collector-cum storage solar water heater is the simplest type of solar water heater available. The solar collector and the insulated storage vessel are combined to form a single unit. The collector-cum storage solar water heater is shown in Figure 1.1. The water stored in the vessel is heated by the solar energy shining through the top cover. However, heat is lost through the surface of the units to the cooler night air. Hence all the hot water has to be used up in the evening.

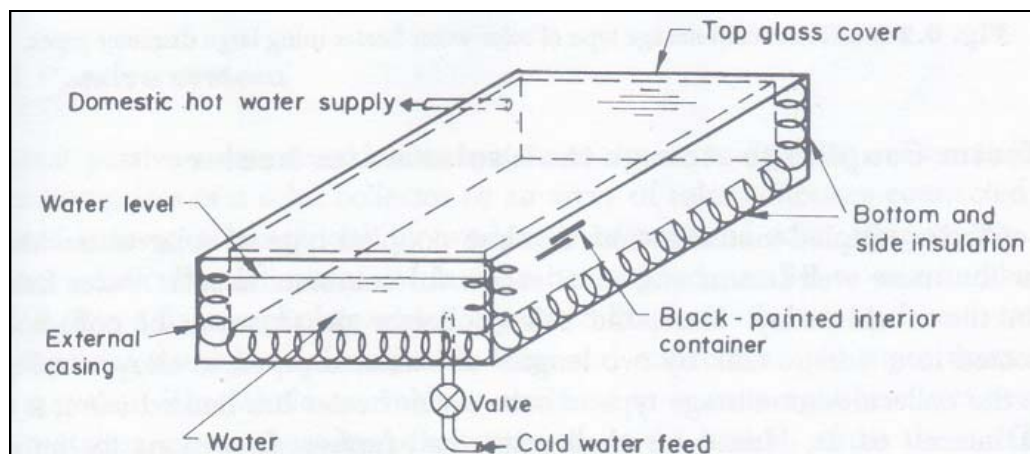


Figure 1.1 : A collector-cum storage type of solar water heater using an open-top vessel

Since the collector-cum storage type of solar water heater has limited use it is not of much interest to us. Hence we shall restrict all further discussions to the closed couple type of solar water heater. The collector-coupled-to storage tank type of solar water heater is the more well-known and most successful commercial solar water heater in use in the world today. The solar collector is connected to a storage tank by two lengths of insulated pipes, as shown in Figure 1.2.

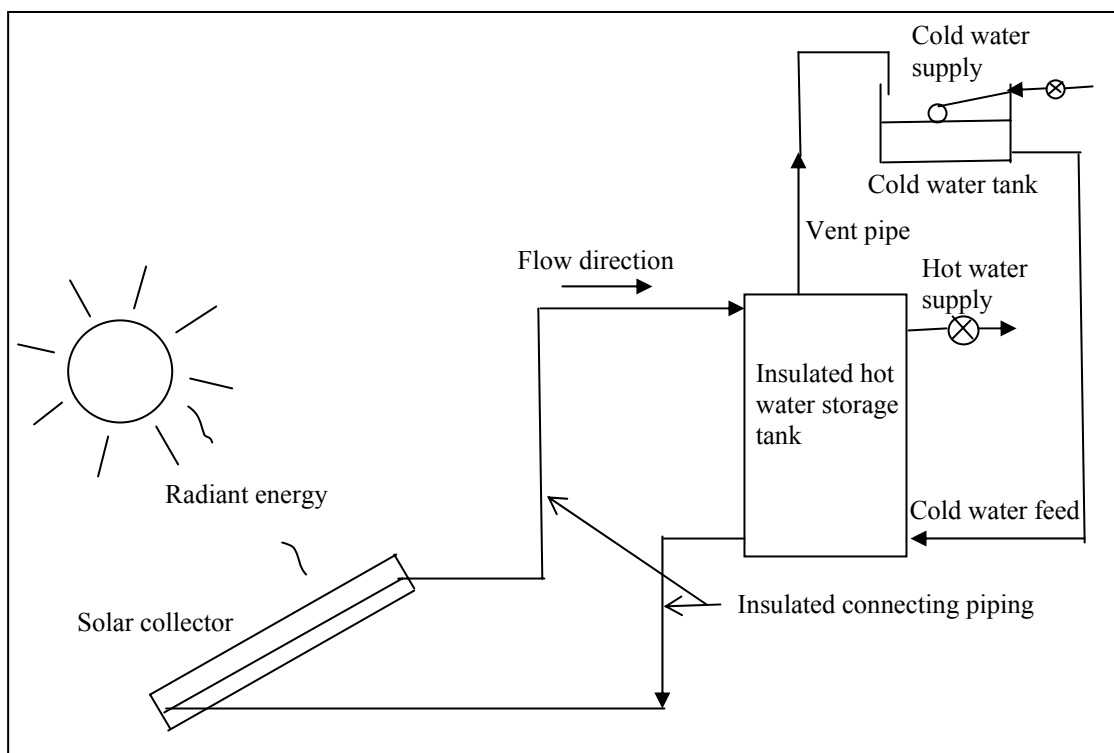


Figure 1.2: A direct, passive and pressurised solar water heater system

Generally, solar water heating system can be conveniently grouped according to the following categorizes ; K.S. Ong (1994),

- Passive (gravity, natural convection, thermosyphon) and active (forced circulation, pumped) system.
- Direct and indirect system.
- Pressurised and non-pressurised (open, draindown) system.

In this project, we are only focusing on the storage tank. Therefore the solar water heating system type is not our main study. In order to study the storage tank passive system type, direct system and pressurised system was selected as shown in Figure 1.2.

The passive solar water heater system comprises of a solar collector or an array of solar collectors connected to an insulated hot water storage tank by two lengths of insulated connecting piping. The solar collector consists of metal tubes aligned parallel to one another and bonded onto a flat metal plate which is painted

black on the top surface to increase its absorptivity towards solar radiation. Inlet and outlet manifold pipes connect the tubes together. The whole assembly is enclosed in a water-tight box and provided with insulation against heat losses around and on the bottom of the box. The top of the box is covered with one or two pieces of thin glass sheets, K.S. Ong (1994).

The storage tank, connecting pipes and collector tubes are completely filled with water. In a passive system, the circulation of the water within the system is by a convection current caused by density differences of the water. Water which is heated in the collector tubes tends to rise to the top of the storage tank. Cold water replaces it from the bottom of the tank. A natural convection current within the system is thus set up. This passive system is also known by various other names such as gravity natural convection, or thermosyphon flow solar water heating system. During the day, as long as there is sufficient sunshine, water will always recirculate in this manner and hot water will accumulate at the top of the storage tank, K.S. Ong (1994).

The details of experimental observation of temperature and flow distribution in a natural circulation solar water heating system and its comparison with the theoretical models can be found in Chuawittayawuth (2001). Natural circulation solar water heating systems are available in varying collector geometries (and materials), storage tank capacities and specifications of individual components. The measured profile of absorber plate temperature around the riser tubes compares well with the theoretical results.

The temperature of the water in the storage tank is higher at the top of the tank than at the bottom owing to thermal stratification. Hence the hot water supply is drawn off from the top or upper portion of the tank. Water drawn off from the top of the tank is replaced by cold supply water piped in from the bottom of the tank. An overhead cold water feed tank supplies cold water to the entire system. The storage tank should be placed at a level higher than the solar collectors. The connecting piping should be kept as short as possible to minimize heat losses. The pipes should be insulated and laid with a continuous upward slope in order to avoid air-locks

which would stop the circulation of water within the system. The collectors too should be sloped to avoid air-locks being formed at the header pipes.

A numerical and an experimental analysis of velocity and temperature fields inside a storage tank submitted to natural convection was investigated by Oliveski et al. (2002). The numerical and experimental results showed that, as time passes, there is a thermal stratification, separating the tank into two different regions: a stratified region at the bottom and another uniform region at the top. The thermal gradients along the radius also have been analysed. At the periphery, thermal gradients are maximum, encompassing the thermal boundary layer. In the center, the thermal radial gradients are no existing.

Water is the most commonly used heat storage medium in solar energy applications because it is cheap and possesses a high specific heat. Most solar heaters operating in the range of 50 to 100⁰C utilizes water for both the heat transfer fluid as well as for storage, , K.S. Ong (1994). The type of storage vessels used in this project is shown in Figure 1.3 which is closed vessel. Storage tanks are usually cylindrical in shape and it can be designed to be installed horizontally or vertically. Horizontal tanks are easier to install on top of roofs although systems operating with horizontal tank are slightly less efficient than those with vertical tanks because of thermal stratification. Vertical tanks encourage thermal stratification which can lead to higher operating system efficiencies.

Thermal stratification occurs in any storage tank when hot fluid is placed above a cooler layer. This arises because of differences in density between the hot and the cold layers. Since the hot fluid has a lower density than the cold fluid, the hot fluid will rise to the top of the tank while the cooler fluid will remain at the bottom. A temperature gradient is thus established throughout the depth of the storage tank. The temperature at the top of the tank will be higher than the temperature at the bottom of the tank. For most solar water heating systems, the difference could vary from about 10 to 30⁰C, depending especially upon the design and locations of the inlet and outlet connections to the solar collector, K.S. Ong (1994).

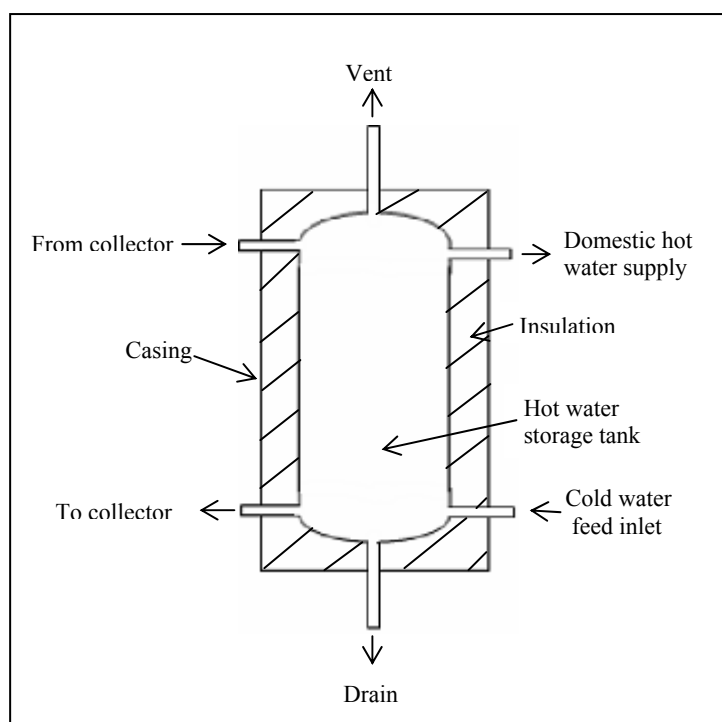


Figure 1.3: Typical storage tank

Oliveski et al. (2003) investigate the comparison between models for the simulation of hot water storage tank. The numerical analysis were performed with two approaches: one using a two-dimensional model in cylindrical coordinates through the finite volume method and another using a one-dimensional model. A turbulence model for low Reynolds numbers was added to the two-dimensional model in mixed convection region. The two dimensional model was experimentally validated and then adopted as reference. Its results were compared to those obtained with one-dimensional models with a good agreement.

Lin and Armfield (1999) investigate numerically the transient process of cooling-down and stratifying an initially homogeneous fluid by natural convection in a vertical circular cylinder. The transient flow patterns are identified by the visualization of the transient evolving processes in the cylinder. The results show that vigorous flow activities concentrate mainly in the vertical thermal boundary layer along the sidewall and in the horizontal region which is the lower part of the domain where the cold intrusion, flow is embedded. The transient flow patterns at the unsteady and quasi-steady stages are analysed, including the activities of the

traveling waves in the vertical thermal boundary layer along the sidewall and the cold intrusions in the horizontal region. A scaling analysis is used to characterize the development of the vertical thermal boundary layer on the sidewall and the stratification in the cylinder. It is found that the numerical solutions agree very well with the scaling results.

Stratification increase collector loop efficiency by enabling only the coldest liquid to be circulated to the collectors for each and every loop of the liquid circulation. Further, stratification leads to higher thermosyphon flow rate because of the increased pressure head causing the liquid circulation. Hence higher operating system efficiencies are obtained. However, stratification offers no advantage in a system where a small storage tank is used in relation to the size of the collectors installed. These systems are usually found in solar heating systems which utilize solar energy as a pre-heater.

Actually the thermal stratification can be improved by adding obstacle in the tank. Effect of using different obstacles on thermal stratification in a cylindrical hot water can be found in Necdet Altuntop et al. (2005). The numerical method is validated using both experimental and numerical results. The results indicate that placing obstacle in the tank provides better thermal stratification compared to the no obstacle case. The obstacle types having gap in the center appear to have better thermal stratification than those having gap near the tank wall. This means they can supply hot water at higher temperatures.

A natural circulation two phase closed thermosyphon flat plate solar water heater has been investigated theoretically under the actual field conditions. Also, the heater design parameters are optimized by simulation program that was verified experimentally, H.M.S Hussein (2002). These parameters include the ratio of storage tank volume to collector area, storage tank dimension ratios and height between the heater storage tank and collector. The computational results indicate that the storage tank volume to collector area ratio and the storage tank dimensions ratio have significant effects on the heater performance, while the height between the heater tank and collector has little effect. The results obtained show that the storage tank

dimensions ratio which is height over diameter (H / D) have significant effects on the heater annual solar fraction and specific useful energy. The optimum value of the storage tank dimensions ratio, H / D is about 1.8.

Storage tanks should be constructed from materials which do not contaminate the water, are non-corrosive, and are stable at operating temperature of up to 100°C . The common materials used for pressurised storage tanks are stainless steel, copper and epoxy-lined mild steel. Pressurised storage tank need to be suitably designed and tested for bursting pressures, K.S. Ong (1994).

Mild steel tanks are often used because they are cheaper than stainless or copper tanks. They can be welded easily and are available in many shapes and sizes. A major disadvantage of steel tanks is that they corrode easily. Therefore steel tanks must be protected both internally as well as externally against corrosion. They must be lined or coated to retard corrosion. Steel tanks are usually epoxy-coated or glass-lined internally. The external surface of steel tanks can be adequately protected by applying conventional paint.

Stainless steel is usually used in place of mild steel tanks. Stainless steel grade 316 is suitable and the inside of the tank should be depassivated after welding. There is no necessity for any further protection both internally as well as externally. Copper tanks are expensive. Usually, thin copper sheet are used to line the insides of thick wall mild steel tanks. Galvanised steel tanks should be avoided in cases where the operating temperature exceeds 65°C due to rapid corrosion occurs above this temperature.

Fiberglass and containers made from plastics like ethylene propylene diene (EDPM) are very resistant to corrosion and easily moulded. They are generally more expensive than mild steel tanks. The major disadvantage of fiberglass and plastic containers is that they are not able to withstand the high operating pressure and temperature. Fiberglass also is susceptible to some organic chemicals. The storage tank insulation is usually protected with an external casing or cladding to protect it

from the environment. The common materials used are usually prepainted mild steel, galvanized steel, aluminium and stainless steel.

Storage heat loss can cause a significant drop in system performance. Hence all hot water storage tanks must be well insulated thermally to prevent heat loss. Insulating material should be selected based on their insulating value and durability. The thickness of insulating material used is governed by the optimum insulation thickness criteria. Optimum insulation thickness is achieved with an appropriate insulating material at a thickness that provides the lowest total life-cycle costs in materials, installation, maintenance and lost energy.

An alternative treatment in lieu of the principle of variational calculus for a certain class of optimization problems can be found in Pramanick (2004). In particular, the optimum distribution of insulating material on one side of a flat plate for minimum heat transfer is sought when the other side is exposed to a laminar forced convection. Both conjugate and non-conjugate formulations of the problem are conceived and closed form solutions are presented. Optimized insulation profile exhibits a category of equipartition principle in some macroscopic domain.

Martin and Dulikravich (2002) developed a computer-automated shape optimization methodology for the purpose of providing internal cooling systems designers the ability to optimize the internal cooling configuration, geometry and heat transfer enhancements for greater cooling efficiency and more durable turbine airfoils. The methodology presents the theory and practical programming requirements for coupling existing computer design and analysis tools together into a new and powerful design system. The goal is to demonstrate the computational advantages of using implicit sensitivity with the boundary element method (BEM) within the system over other more brute force method.

Some common insulations material used are fiberglass wool, mineral rockwool, expanded polystyrene and polyurethane foamed-in-situ. The insulation values of these materials are given in Table 1. The material with the highest thermal

conductivity / weight ratio is polyurethane. Polystyrene should be used over a thin layer of fiberglass wool insulation because it breaks down at around 80°C . The use of polystyrene and polyurethane may be prohibited in certain countries because of smoke and fire regulations. Polyurethane breaks down at temperatures around 80°C , K.S. Ong (1994).

The temperature distribution through a cross-section of an insulated storage tank containing water at a uniform temperature is shown in Figure 1.4. The inside of the tank is shown covered with a layer of scale. The outside of the tank is in contact with the surrounding ambient air. For clean thin wall metal tanks clad with a thin metal sheet, the heat loss from the tank to the ambient is mainly dominated by the thermal insulation and the film of air in contact with the surface.

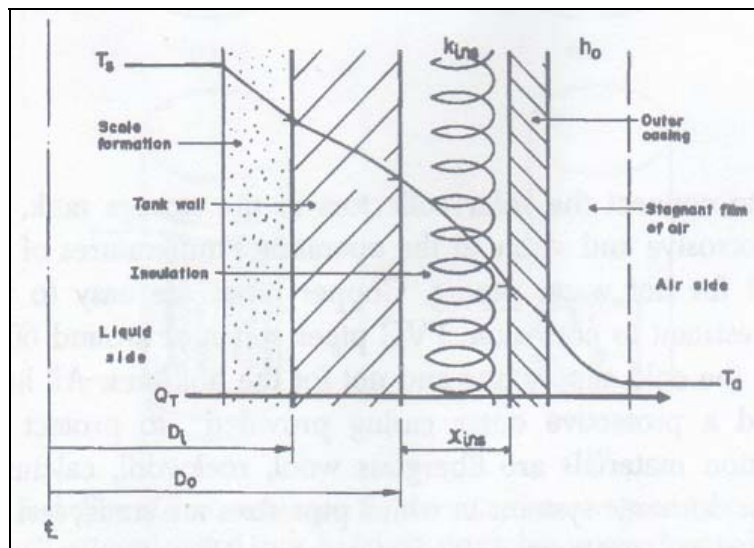


Figure 1.4 : Section through insulated storage tank

The most convenient method of computing heat loss from the exposed surface of the insulation to the ambient air is to use a thermal resistance coefficient h_o that represents both convection and conduction heat transfer. The heat loss per unit surface area of storage tank may be represented by

$$Q_s = U_s A_{ins} (T_s - T_a)$$

where A_{ins} is the total surface area of the insulated tank, T_a is the temperature of the environment in which the storage tank is kept. U_s is the overall heat transfer coefficient which will be derived later on.

Table 1.1 : Thermal properties of some insulating materials *

Material	Density (kg/m ³)	Thermal conductivity (W/m K)	Maximum temperature (°C)
Asbestos cement board	945	0.19	
Brick (red)	1760	0.65	
Calcium silicate	100 - 230	0.040 - 0.055	1000
Cardboard	-	0.07	
Coconut fiber husk	48	0.053	
Concrete (1:2:4 mix)	1680	0.40	
Cork board	144	0.042	
Felt wool	136 - 168	0.039	
Fiberglass wool	12 - 80	0.032 - 0.040	450
Glass (soda-lime)	2470	1.00	
Gypsum board	880	0.17	
Mineral rockwool	32 - 176	0.032 - 0.035	650
Paper	1090	0.14	
Phenolic foam	32	0.10	150
Plywood	530	0.14	
Polystyrene foam	16	0.020 - 0.039	80
Polyurethane foam (rigid)	24	0.016 - 0.025	80
(flexible)	40	0.035 - 0.039	80
Rubber (sheet)	930	0.16	
Sand (building)	1500	0.30	
Sawdust	200	0.059	
Wood (hard)	370 - 1100	0.110 - 0.255	

*Source ; Solar Water Heaters Engineering and Applications, K.S. Ong (1994)

The storage tank insulation is usually protected with an external casing or cladding to protect it from the environment. The common materials used are usually pre-painted mild steel, galvanized steel, aluminium and stainless steel. All

penetrations for pipes, etc., must be adequately water-proofed in order to preserve the waterproofing protection.

1.3 Current Problem

Malaysia is a small nation in South East Asia situated a few degrees north of the equator. The country enjoys a near uniform climate throughout the year with hot sunny days punctuated with occasional showers in the evening. The tropical nights are typically warm. Daily ambient temperatures ranges from about 23 to 35°C. The average daily total solar radiation amounts to about 4500 Wh/m². Apart from high humidity and large patches of intermittent cloud cover, Malaysian climatic conditions favour the use of solar energy for water heating, K.S Ong (1994).

The average daily domestic consumption is about 45 l per person of hot water at 60°C for showering purpose in the evening and early morning. Normally it is about 30 l in the evening and 15 l at early morning. This 60°C water would be mixed with cold water at the shower head. The temperature of the cold water supply is generally about 27°C and the mixed temperature at the point of supply is about 32°C. Long baths would require about 2 to 3 times this amount of hot water but in this study, a consumption figure of 45 l per person would be assumed. Thus, for an average household of 4 to 6 person, the total hot water consumption is about 180 to 270 l. Keeping in mind that as the solar-heated water is drawn off from the top of the storage tank, the cold water entering from the bottom of the tank mixes with the rest of the hot water in the tank and reduces the stored water temperature, K.S Ong (1994).

After some time, the temperature of the water in the tank would be lowered to such a level that the stored water would only be luke warm and be inadequate for shower. As a general rule, about 80% of the hot water in a hot water storage tank can be assumed to be practically utilized for calculation purposes because of this dilution factor. As a result, storage tank capacities of about 225 to 338 l are required. Most of

the solar hot waters marketed nowadays have standard storage tank capacities ranging from 136 to 300 l. Therefore, storage tanks should be well-insulated to minimize heat losses. The insulating material should not degrade at the operating temperature of the storage tank which is typically around 80°C, K.S Ong (1994).

The intermittency of the sun shines such as cloudy or rainy day and at night will cause the hot water temperature in the storage tank drop. The main purpose of solar water heater system is to provide hot water especially during cold weather. Therefore, if the water storage tank is unable to supply hot water during the cold day the solar water heater system is less useful. Hence it is important to design the water storage tanks that able to maintain the hot water temperature in the storage tank for certain duration. As described earlier the water in the storage tank is hot at the top and cold at the bottom. The hot water temperature will transfer the heat to the cold water slowly and it will show temperature distribution in the storage tank. The effect of the velocity of the water flowing in a thermosyphon flow solar water heater and heat losses need to be considered as well.

Actually it is hard to determine the thickness of the insulator since the temperature varies along the vertical axis of the storage tank and the profile is change as time increase. It can be determine by simply choosing the average temperature at the initial stage between the top and bottom of the water temperature and then determine the thickness of insulator but it may cause the tank is over insulated or less insulated. Therefore the temperature distributions in the tank need to be analyzed in order to obtain more accurate average temperature for certain duration.

Currently the existing solar water heater has good efficiency but the demand of the solar water heater in Malaysia is still low because of the price. Both criteria, efficiency and cost are the main factors that need to be considered before buying the system. Therefore it is necessary to produce solar water heater system with low cost but at high efficiency. The design of an optimum solar water heater storage tank can be obtained by optimization process.

1.4 Objectives and Scopes of Project

The final objective of this project is to obtain the optimum cost of solar water heater storage tank. The cost optimization process will be done with some relevant constraint and necessary assumption. The main objectives of the project can be outlined as per below;

- 1) To obtain the objective function for cost optimization, volume constraint and heat losses constraints.
- 2) To investigate the temperature distribution along the vertical axis of hot water storage tank in order to obtain the average temperature.
- 3) To obtain the minimum cost of solar water heater storage tank.

The scopes of this projects are :

- 1) The analysis will be done on vertical solar water heater storage tank.
- 2) The thermal analysis will be considered for 10 hours duration, start from 8.00 a.m to 6.00 p.m.
- 3) Lagrange multiplier method will be used to solve the cost optimization.
- 4) The temperature distribution in the storage tank and the unknowns (independent variables and Lagrange multipliers) will be solved by numerical method.